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On the Transient Response and Backscatter Properties of Linear Antennas

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CONTENTS

FOREWORD

Three papers entitled "Transient Response of a Linear Antenna to Pulse Voltage Excitation," "Transient Terminal and Field Properties of Linear Antennas," and "Backscattered Field of a Linear Antenna and a Two Element Array" constitute the three chapters of this report. They represent the approach taken by Dr. Sandler to the analysis of scattering from an array of linear antennas. The first paper develops an expression for the far field of a linear antenna excited by a pulse. A simple approximation for the current distribution on the antenna is assumed in this derivation. The second paper deals with essentially the same problem only from a point of view which is analogous to traveling waves on a transmission line. The concepts developed in these two papers have been incorporated into the third to investigate the backscatter properties of both a linear antenna and an array composed of two linear elements.

A parallel effort by Dr. O. D. Sledge of NRL is currently nearing completion. The first phase of this work has been published as NRL Report 6691, "The Scattering of a Plane Electromagnetic Wave by a Linear Array of Center-Loaded Cylinders." The approach has been to investigate the array backscatter for steady state excitation. It is planned to integrate these results over a broad band of specific frequencies to obtain the transient response.

These two efforts represent a cooperative approach to the problem in that the same goals are desired and similar models have been chosen. Because of a lack of precedence in this particular subject the problem was approached in different ways, the basic difference being the manner chosen to obtain the current distribution on the elements of the array. It is expected that the two approaches will complement one another to provide a better understanding of the scattering from arrays of linear elements.

PROBLEM STATUS

This is a final report on one phase of the problem; work on the problem is continuing.

AUTHORIZATION

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ON THE TRANSIENT RESPONSE AND BACKSCATTER PROPERTIES OF LINEAR ANTENNAS

CHAPTER 1

TRANSIENT RESPONSE OF A LINEAR ANTENNA TO PULSE VOLTAGE EXCITATION

One method of finding the transient response of a single antenna or an array is through the reciprocity relation (1). The reciprocity concerns the relation between the far zone electric field and the receiving current on a linear antenna. Consider the linear cylindrical antenna of length $2\hat{h}$ shown in Fig. 1. The far zone field $E_\theta^r(\theta)$ is

$$E_\theta^r(\theta) = \frac{j\zeta_0 I_0}{2\pi} F(\theta, \beta_0 h) \frac{e^{-j\beta_0 r_1}}{r_1}, \tag{1}$$

where

$$\zeta_0 = 120\pi \text{ ohms}, \tag{2}$$

$$I_0 = \frac{V_0}{Z_0 + Z_g} = \frac{V_0 e^{-j\omega t}}{Z_0 + Z_g}, \tag{3}$$

and

$$F(\theta, \beta_0 h) = \text{field factor}. \tag{4}$$

With Eq. (3) substituted in Eq. (1) the retarded electric field is expressible as

$$E_\theta^r(\theta) = \frac{jV_0\zeta_0}{(Z_0 + Z_g)2\pi} F(\theta, \beta_0 h) \frac{e^{j\omega[t-(r_1/c)]}}{r_1}. \tag{5}$$

Thevenin's theorem may be used to find the current in a load resistor which is connected to a receiving antenna. For the receiving antenna of Fig. 2, the open circuit voltage V_{oc} is

$$V_{oc} = -2h_e(\theta)\, E_0\, e^{j\omega[t-(r_2/c)]} \cos\psi. \tag{6}$$

By Thevenin's theory the current in a load resistor connected to the antenna, I_L, is

$$I_L = \frac{V_{oc}}{Z_0 + Z_L} = \frac{-2h_e(\theta)}{Z_0 + Z_L} E_0\, e^{j\omega[t-(r_2/c)]} \cos\psi. \tag{7}$$

The effective length of the receiving antenna is related to the field factor of a transmitting antenna through the reciprocity relation; thus

$$F(\theta, \beta h) = \beta_0 h_e(\theta, \beta h). \tag{8}$$

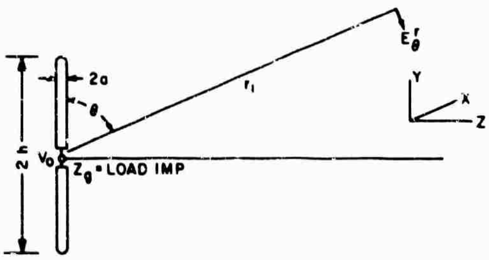

Fig. 1 - Driven antenna

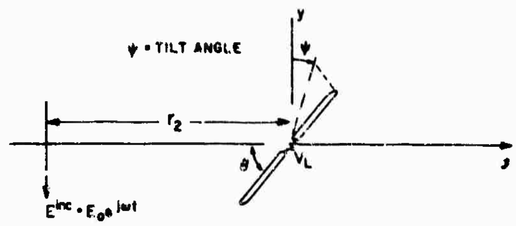

Fig. 2 - Receiving antenna

Equation (8) expresses the idea that the transmitting and receiving patterns of antennas are identical. When Eq. (7) with Eq. (8) is differentiated with respect to time, the resulting relation is found to be proportional to the far zone electrical field.

The transient response of a single antenna can be considered as a superposition of the steady state components which make up the transient signal. For example, the Fourier representation of a voltage step $V(t)$ is

$$V(t) = \frac{V_0}{2} + \frac{V_0}{\pi} \int_0^\infty \frac{\sin \omega t}{\omega}\, d\omega . \tag{9}$$

A pulse length t_0 can be represented by two step functions displaced in time; thus

$$V_p(t) = V(t) - V(t - t_0) , \tag{10}$$

where $V(t)$ is given by Eq. (9).

To get the response of an antenna to a pulse it is only necessary to compute the response to a step function and apply superposition. The basic expression for the step function transient response of a broadside illuminated antenna is given by Schmitt and is

$$E_\theta^r(t) = \frac{V_0 60}{r_1 \pi} \left[\int_0^\infty A(\beta h) \cos \beta h \left(\frac{ct_1}{h}\right) d\beta h + \int_0^\infty B(\beta h) \sin \beta h \left(\frac{ct_1}{h}\right) d\beta h \right] , \tag{11}$$

where

$$A(\beta h) = Im \frac{j\beta h_e(\theta)}{(Z_0 + Z_g)\beta h} , \tag{12a}$$

$$B(\beta h) = Re \frac{j\beta h_e(\theta)}{(Z_0 + Z_g)\beta h} , \tag{12b}$$

and

$$t_1 = t - \frac{r_1}{c} . \tag{12c}$$

To evaluate Eq. (11) with Eqs. (12) the values of effective length and driving point impedance must be known over a large frequency range. The range is determined by the relative contribution of the particular frequency terms to the integral.

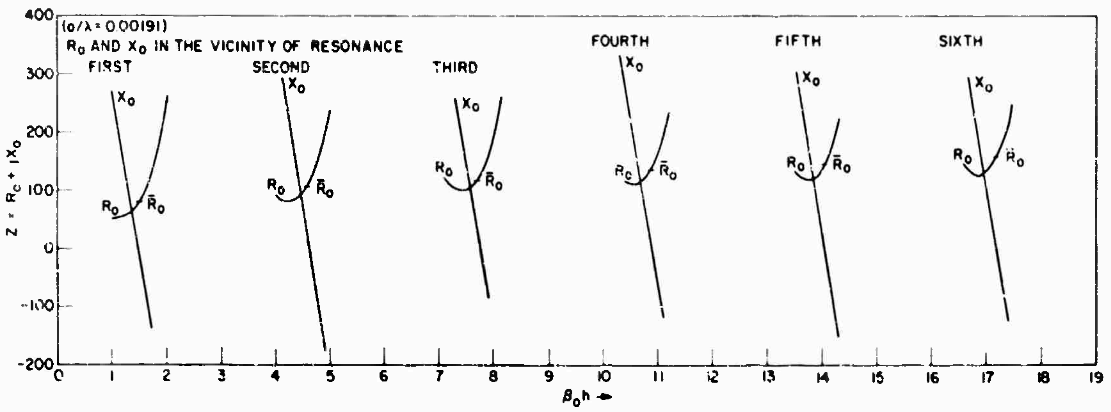

Fig. 3 - Driving point impedance near resonance

Based on published impedance data for long antennas, given by Wu (2), the driving point impedance is plotted in the vicinity of the resonant points in Fig. 3. Note that in the vicinity of resonance the resistance is moderately constant and the reactance has almost a linear slope. Thus in the vicinity of the first resonance the following representation is possible:

$$Z_0 = \bar{R}_0 + j(\bar{X}_0 - m\beta_0 h) \qquad \text{(first resonance)}, \qquad (13)$$

where

$\bar{R}_0 \approx 125\ \Omega,$

$\bar{X}_0 \approx 750\ \Omega,$

$m \approx 500\ .$

At the second resonance

$$Z_0 = \bar{R}_0 + j(\bar{X}_0 + \pi\bar{X}_0 - m\beta h) \qquad \text{(second resonance)}. \qquad (14)$$

In general

$$Z_0 \approx \bar{R}_0 + j[(1 - \pi + n\pi)\bar{X}_0 - m\beta h], \quad n = 1, 2, 3, \ldots\ . \qquad (15)$$

A Fourier trigonometric representation for Eq. (15) is

$$Z_0 = \bar{R}_0 + j\sum_{\nu=1}^{\infty} \bar{X}_\nu \cos \nu\beta h, \quad \bar{X}_\nu = \frac{m}{\pi\nu^2}(\cos \nu\pi - 1)\ . \qquad (16a)$$

or

$$Z_0 = \bar{R}_0 + j\bar{X}_1 \cos \beta h + \ldots\ . \qquad (16b)$$

Here it is to be noted that the major contributions to the integral occur in the vicinity of resonance. A continuation for functions can be made in the region away from resonance, provided there is a negligible contribution to the integral. The zeroth-order effective length at broadside incidence is

$$\beta h_e\left(\frac{\pi}{2}\right) = \tan\frac{\beta h}{2} . \tag{17}$$

The simplified form for the transient ele ic field is

$$E_0^r(t) = \frac{60V_0}{r_1}(I_1 + jI_2) , \tag{18}$$

where

$$I_1 = \bar{R}_0 \int_0^\infty \frac{\tan\frac{x}{2}}{\bar{R}_0^2 + \bar{X}_1^2 \cos x}\left(\frac{\cos \alpha x}{x}\right) dx , \tag{19}$$

$$I_2 = -\bar{X}_0 \int_0^\infty \frac{\tan\frac{x}{2}\cos x}{\bar{R}_0^2 + \bar{X}_1^2 \cos^2 x}\left(\frac{\sin \alpha x}{x}\right) dx , \tag{20}$$

and

$$\alpha = \frac{ct_1}{h} = \frac{c}{h}\left(t - \frac{r_1}{c}\right) . \tag{21}$$

The parameter α in Eq. (21) measures the time it takes for an electromagnetic wave to travel from the center to the end of the antenna.

Since near resonance

$$\left(\beta h = n\frac{\pi}{2} , \ n \text{ odd}\right) \tan\frac{x}{2} = \sin x ,$$

then I_1 becomes

$$I_1 \approx \bar{R}_0 \int_0^\infty \frac{\sin x}{\bar{R}_0^2 + \bar{X}_1^2 \cos^2 x}\left(\frac{\cos \alpha x}{x}\right) dx . \tag{22}$$

Before Eq. (22) is evaluated the properties of the following integral will be investigated:

$$I_3 = \int_0^\infty \sin px \left(\frac{\cos qx}{x}\right) dx = \frac{\pi}{2} , \quad 0 < q < p , \tag{23a}$$

$$= \frac{\pi}{4} , \quad 0 < p = q , \tag{23b}$$

$$= 0 , \quad 0 < p < q . \tag{23c}$$

When the first term of the integral in Eq. (22) is expanded in a Fourier series, an infinite number of integrals resembling Eq. (23) will appear, each with a different coefficient and integer value of p. The integral in Eq. (23) represents a series of step functions beginning at $q = 0$ and ending at $q = p$. When a series of alternating step functions of alternating sign are added, a series of pulses can be produced. Figure 4 shows an example of such a series of pulses. The integral I_1 may be placed in the following form after the trigonometric expansion:

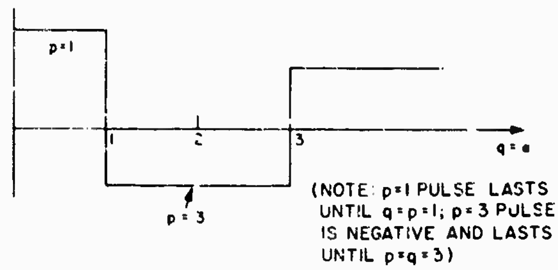

Fig. 4 - Train of pulses produced by Eq. (23)

$$I_1 \approx \bar{R}_0 \sum_{n=1}^{\infty}{}' b_n \int_0^{\infty} \sin nx \left(\frac{\cos \alpha x}{x}\right) dx . \tag{24}$$

The coefficients b_n are evaluated, in the Appendix, in terms of the general integral I_a, where

$$I_a = \frac{1}{\pi} \int_0^{\pi} \frac{\cos ax}{a^2 + \cos^2 x} dx . \tag{25}$$

Expressed in terms of I_a, the first few coefficients b_n are

$$b_1 = \frac{I_0 - I_2}{2}, \qquad b_2 = 0,$$

$$b_3 = \frac{I_2 - I_4}{2}, \qquad b_4 = 0,$$

$$b_5 = \frac{3}{2}(I_0 - I_2) + \frac{1}{2}(I_4 - I_6),$$

where

$$I_a = \frac{(Z_1)^{a/2} + (Z_1)^{-a/2}}{2a\sqrt{1 + a^2}} + Res\ (z = 0) , \tag{26}$$

$$Z_1 = -(1 - 2a^2) + 2a\sqrt{1 + a^2} \approx -1 + 2a , \quad a^2 \ll 1 , \tag{27}$$

in which $\alpha = 2$, $Res\ (z = 0) = 1$; $\alpha = 4$, $Res\ (z = 0) = -1 + 2a^2$. Note that when $(\bar{R}_0/\bar{X}_1)^2 \ll 1$, $b_n \approx (-1)^n/\alpha$, n odd. This case is shown in Fig. 5. Note that all the pulses are the same height. Note also that the first pulse is of length $\alpha = 1$, and all other pulses have a length equal to $\alpha = 2$. Actually, when the "damping terms" in $\alpha = \bar{R}_0/\bar{X}_1$ are added, the pulse heights are seen to decrease for increasing time. The integral I_2 in the far field expression can be placed in the form

$$I_2 = -\frac{1}{2\bar{X}} \sum_{n\ \mathrm{even}} \bar{b}_n \int_0^{\infty} \sin nx \left(\frac{\sin \alpha x}{x}\right) dx . \tag{28}$$

After integration, Eq. (28) becomes

$$I_2 = -\frac{1}{2\bar{X}} \sum_{n\ \mathrm{even}} \bar{b}_n \ln \sqrt{\frac{n + \alpha}{n - \alpha}} , \quad n \neq \alpha , \tag{29a}$$

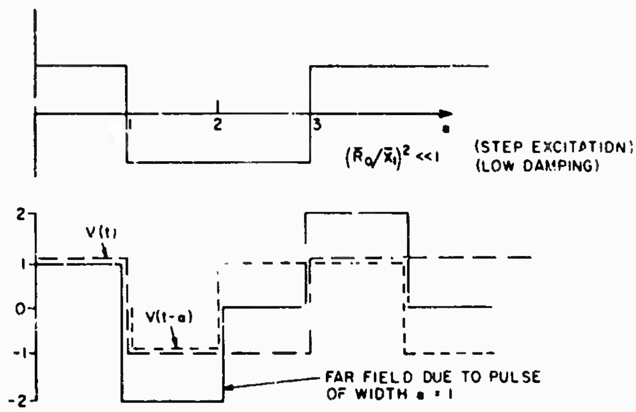

Fig. 5 - Step response (top) and pulse response obtained by combining two step responses (bottom)

$$I_2 = -\frac{1}{2\bar{X}} \sum_{n \text{ even}} \bar{b}_n \frac{1}{2} Cin\, x. \qquad n = \alpha. \tag{29b}$$

The main contribution of the I_2 integral to the field is a spiked pulse at α = 2, 4, 6, etc. This spike is not included in Fig. 5.

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1. Schmitt, H.J., "Transients in Cylindrical Antennae," Proc. Inst. Elec. Engrs. (London) 107 (Part C) 292-298, April 1960

2. Wu, T.T., "Theory of the Dipole Antenna and the Two-Wire Transmission Line," J. Mathematical Phys. 2:550 (1961)

Appendix

EVALUATION OF THE COEFFICIENTS b_n

Consider the general integral I_a, where

$$I_a = \frac{1}{\pi}\int_0^{\pi} \frac{\cos \alpha\theta}{a^2 + \cos^2\theta}\, d\theta = \frac{1}{\pi}\int_0^{\pi} \frac{2\cos \alpha\theta}{2a^2 + 1 + \cos 2\theta}\, d\theta\,, \tag{A1}$$

where $a = \bar{a}_0/\bar{X}_1$. With the substitution $\phi = 2\theta$, Eq. (A1) becomes

$$I_a = \frac{1}{\pi}\int_0^{2\pi} \frac{\cos \frac{\alpha\phi}{2}}{1 + 2a^2 + \cos\phi}\, d\phi\,. \tag{A2}$$

Equation (A2) may be evaluated by complex variable theory by tegrating around the unit circle After the substitution $z = e^{i\phi}$, Eq. (A2) is reduced to the contour integral

$$I_a = \frac{-i}{\pi}\oint_c \frac{z^{\alpha/2} + z^{-\alpha/2}}{2z\,(1 + 2a^2) + 1 + z^2}\,, \tag{A3}$$

where c is the unit circle. If α is even, there are no branch points and the poles are located at

$$z_0 = 0 \quad \text{and} \quad z_{1,2} = -(1 + 2a^2) \pm 2a\sqrt{1 + a^2}\,. \tag{A4}$$

The poles within the unit circle are at $z_0 = 0$ and $z_1 = -(1 + 2a^2) \pm 2a\sqrt{1 + a^2}$. The pole at $z_0 = 0$ is of order $(\alpha/2)$. The integral is given by multiplying the sum of the residues by $2\pi i$; thus

$$I_a = 2\pi i \sum_i Res = 2\pi i\, \frac{(z)^{\alpha/2} + (z)^{-\alpha/2}}{4a\sqrt{1 + a^2}} + Res\,(z = 0)\,. \tag{A5}$$

The residue at 0 depends on the order of the pole at $z = 0$. In general the residue is given by

$$Res = \frac{1}{(n-1)!}\,\frac{d^{n-1}}{dz^{n-1}}\left[(z - z_n)^n\, \frac{f(z)}{g(z)}\right], \quad z = 0\,, \tag{A6}$$

where

$$n = a/2, \qquad z_n = 0\,,$$

and

$$\frac{f(z)}{g(z)} = \frac{z^{\alpha/2} + z^{-\alpha/2}}{z^2 + 2z\,(1 + a^2) + 1}$$

$$= \frac{1}{z^{\alpha/2}} \;\frac{z + 1}{z^2 + 2z\,(1 + 2a^2) + 1} \,. \tag{A7}$$

The first three residues are evaluated as follows:

for $\alpha = 2$**,**

$$n = 1: \quad Res = 1 \,,$$

for $\alpha = 4$**,**

$$n = 2: \quad Res = -1 + 2a^2 \,,$$

and for $\alpha = 6$**,**

$$n = 3: \quad Res = 2 + 2\,(1 + 2a^2)\,[1 - 2\,(1 + a^2)] \,.$$

CHAPTER 2

TRANSIENT TERMINAL AND FIELD PROPERTIES OF LINEAR ANTENNAS

INTRODUCTION

The transient performance of antennas has received much less theoretical and experimental attention than that of the steady state. Notable contributions to the understanding of the transient performance of antennas have been made by Schmitt (1), Schmitt and King (2), Schmitt, Harrison, and Williams (3), Tseng and Cheng (4), and Bulgakov, Busev, and Rysakov (5).

Schmitt, numerically and experimentally, investigated the step function response of a single dipole. Using a limited range of driving point impedance data, Schmitt found the antenna field and base current for a step voltage excitation. The results agreed in essential parts with experiment. A simple qualitative picture was given to explain the time variation of the far zone electric field. The physical picture corresponding to the current and charges on a short-circuited transmission line excited by a step voltage was used for a model of the radiation processes of an antenna.

Schmitt and King explained the early time response of a linear antenna. The time interval examined was less than the time it took for the first pulse to return to the driving point. A reflection coefficient was defined based on an average value for the range of steady state frequencies contained in the transient. The impedance used here was that of an infinite antenna.

Schmitt, Harrison, and Williams used large steady state ranges of the effective length and driving point impedance to predict the transient response of a single dipole. Tseng and Cheng used simple assumed current distributions to produce a simplified analytical determination of the transient characteristics of antennas and arrays. Bulgakov, Busev, and Rysakov pointed out the errors inherent in simplified analyses of the transient performance of antennas. They also examined the field structure for a cylindrical antenna under transient conditions.

It is the purpose of this chapter to (a) place the transient performance of a linear antenna using the transmission line pulse analogy on a stronger theoretical base, and (b) use the superposition principle to include the actual impedance properties of the antenna.

TRANSMISSION LINE ANALOGY

Consider the transmission line of Fig. 1 of length l with characteristic impedance Z_0, driving point impedance Z_1, and load impedance Z_2. The traveling-wave current on such a lossless line is approximately

$$I(Z,\omega) = I_0 \left[e^{-j\omega Z/v} - r_2 e^{-j\omega(2l-z)/v} + r_1 r_2 e^{-j\omega(2l+z)/v} + \ldots\right] . \tag{1}$$

The reflection coefficients r_1 and r_2 are

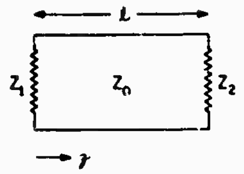


Fig. 1 - Lossless transmission line of length l

$$r_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0} \tag{2a}$$

and

$$r_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0} \,. \tag{2b}$$

The simple interpretation of Eq. (1) is that of a traveling wave of constant phase velocity (except at the ends), which is continually reflected at $z = 0$ and $z = l$. The time response of Eq. (1) for a step function of current is

$$I(Z_1, t) = I_0 \left\{ H\left[t - \frac{z}{v}\right]_0^l - r_2 H\left[t - \frac{2l - z}{v}\right]_l^0 + r_1 r_2 H\left[t - \frac{2l + z}{v}\right]_0^l + \dots \right\}, \tag{3}$$

where $H(t) = 0$ for $t < 0$, $H(t) = 1$ for $t \geq 0$, and r_1 and r_2 are constants. In Eq. (3) $H[t - z/v]_0^l$ signifies that the step function exists for all times t, where $0 \leq t \leq l/v$ or

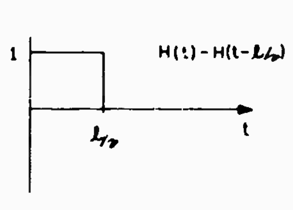
$$H\left[t - \frac{z}{v}\right]_0^l = H[t] - H\left[t - \frac{l}{v}\right], \tag{4}$$

$$H\left[t - \frac{2l - z}{v}\right]_l^0 = H\left[t - \frac{l}{v}\right] - H\left[t - \frac{2l}{v}\right]. \tag{5}$$

etc. Equation (4) expresses the fact that the first traveling wave exists only over the time it takes to reach the load impedance at $z = l$ (see Fig. 2).

Consider now a center-fed dipole of length $2h$ shown in Fig. 3. There are, in this case, two traveling waves originating at $z = 0$. Both waves travel outward toward the ends at $z = \pm h$ and are reflected by the open circuit ($r_2 = 1$). After traveling back to $z = 0$, the waves are partially reflected back and transmitted to the opposite side of the antenna. The paths of general rays are depicted in Fig. 4. The traveling wave forms which correspond to Fig. 4 are, for $z > 0$,

Fig. 2 - Rectangular pulse of length l/v



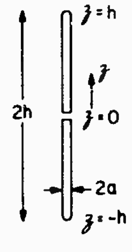


Fig. 3 - Center-fed dipole of length $2h$

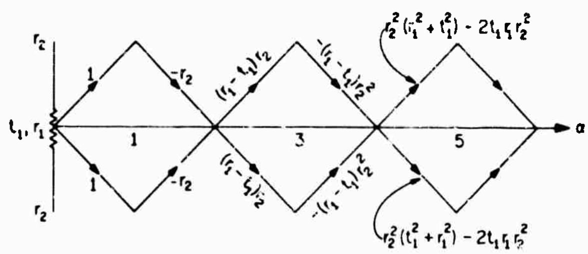


Fig. 4 - General form of traveling waves

$$I(z,\omega) = I_0\left[e^{-j(\omega/c)z}\Big|_0^h - e^{-j(\omega/c)(2h-z)}\Big|_h^0 + (r_1 - t_1)e^{-j(\omega/c)(2h+z)}\Big|_0^h\right.$$

$$\left. - (r_1 - t_1)e^{-j(\omega/c)(4h-z)}\Big|_h^0 + \ldots\right], \tag{6}$$

where t_1 = transmission coefficients at $z = 0$. In Eq. (6) $e^{-j(\omega/c)z}\Big|_0^h$ signifies that the wave exists only over the time that it takes the wave to travel from $z = 0$ to $z = h$. The waves in Fig. 4 for $z < 0$ are

$$I(z,\omega) = I_0\left[e^{+j(\omega/c)z}\Big|_0^{-h} - e^{-j(\omega/c)(2h+z)}\Big|_{-h}^0 + (r_1 - t_1)e^{-j(\omega/c)(2h-z)}\Big|_0^{-h}\right.$$

$$\left. - (r_1 - t_1)e^{j(\omega/c)(4h+z)}\Big|_{-h}^0 + \ldots\right]. \tag{7}$$

The radiated electric field E_θ^r can now be computed based on the current form given by Eqs. (6) and (7). Since the antenna current is symmetrical with respect to z, either formula may be used in the general relation for F_θ^r given by

$$E_\theta^r = j\,\frac{\zeta_0}{2\pi}\,I_z(0)\,\frac{e^{-j\omega[t-(r/c)]}}{r}\,F_0(\theta,\beta_0 h), \tag{8}$$

where

$$F_0(\theta,\beta_0 h) = \frac{(\omega/c)\sin\theta}{I_z(0)}\int_{-h}^{h} I_z(z')e^{j(\omega/c)z'\cos\theta}\,dz', \tag{9}$$

$\zeta_0 = 120\pi$ ohms, and $\beta_0 = \omega/c$. If the current is a step function, then $I_z(t)$ has the representation

$$I_z(t) = \frac{I_0}{2} + \frac{I_0}{\pi}\int_0^\infty \frac{\sin\omega t}{\omega}\,d\omega. \tag{10}$$

The time variation of the far-zone electric field follows from an integration of the steady state relation of Eq. (8) with Eq. (10). For the first traveling wave

$$E_\theta^r(t) = \frac{\zeta_0}{2\pi}\,\frac{I_z(0)}{r}\,Im\int_0^\infty jF_0(\theta,\beta_0 h)\,\frac{e^{j\omega t}}{\omega}\,d\omega \tag{11a}$$

$$E_\theta^r(t) = \frac{\zeta_0}{2\pi}\frac{I_z(0)}{r}\int_{-h}^{h} dz' \frac{\sin\vartheta}{c}\, Im \int_0^\infty jRec\,\frac{z'}{h}\, e^{j\omega[(z'/c)(\cos\theta-1)+t]}\, d\omega\,, \tag{11b}$$

where

$$Im = \text{imaginary part}$$

$$Rec\,\frac{z}{h} = 0\,,\quad |z| > h$$

$$= 1\,,\quad |z| \le h\,.$$

Since the delta function $\delta(t)$ is given by

$$\delta(t) = \frac{1}{\pi}\int_0^\infty \cos\omega t\, d\omega\,, \tag{12}$$

Equation (11b) becomes

$$E_\theta^r(t) = \frac{60 I_z(0)}{r}\int_{-h}^{h} Rec\left(\frac{z'}{h}\right)\delta\left[t - \frac{z'}{c}(1-\cos\theta)\right] dz'\,. \tag{13}$$

The integral in Eq. (13) is easily evaluated from the definition of the delta function $\delta(t)$ and $Rec(z/h)$. In general,

$$\int_{-h}^{h} Rec\left(\frac{z'}{h}\right)\delta(t-\gamma)\, dz' = 1,\quad t \le \gamma(h)\,, \tag{14a}$$

$$= 0,\quad t > \gamma(h)\,. \tag{14b}$$

With Eq. (14) in Eq. (13) the radiated electric field for the first traveling wave is

$$E_\theta^r(t) = \frac{60 I_z(0)}{r}\left\{H(t) - H\left[t - \frac{h}{c}(1-\cos\theta)\right]\right\} \tag{15a}$$

$$= \frac{60 I_z(0)}{r}\,[H(t) - H(t-a)]\,, \tag{15b}$$

where

$$a = \frac{h}{c}(1-\cos\theta) \tag{16}$$

Equation (15) says that the first traveling wave produces a far field that starts at $a = 0$ and ends at $a = h(1-\cos\theta)/c$. A generalization of this result for the current of Eq. (6) gives

$$E_\theta^r(t) = \frac{120 I_z(0)}{r}\Big\{[H(t) - H(t-a)] - [H(t-a) - H(t-2a)]$$

$$+ (r_1 - t)\,[H(t-2a) - H(t-3a)] - (r_1 - t)\,[H(t-3a) - H(t-4a)] + \ldots\Big\}\,. \tag{17}$$

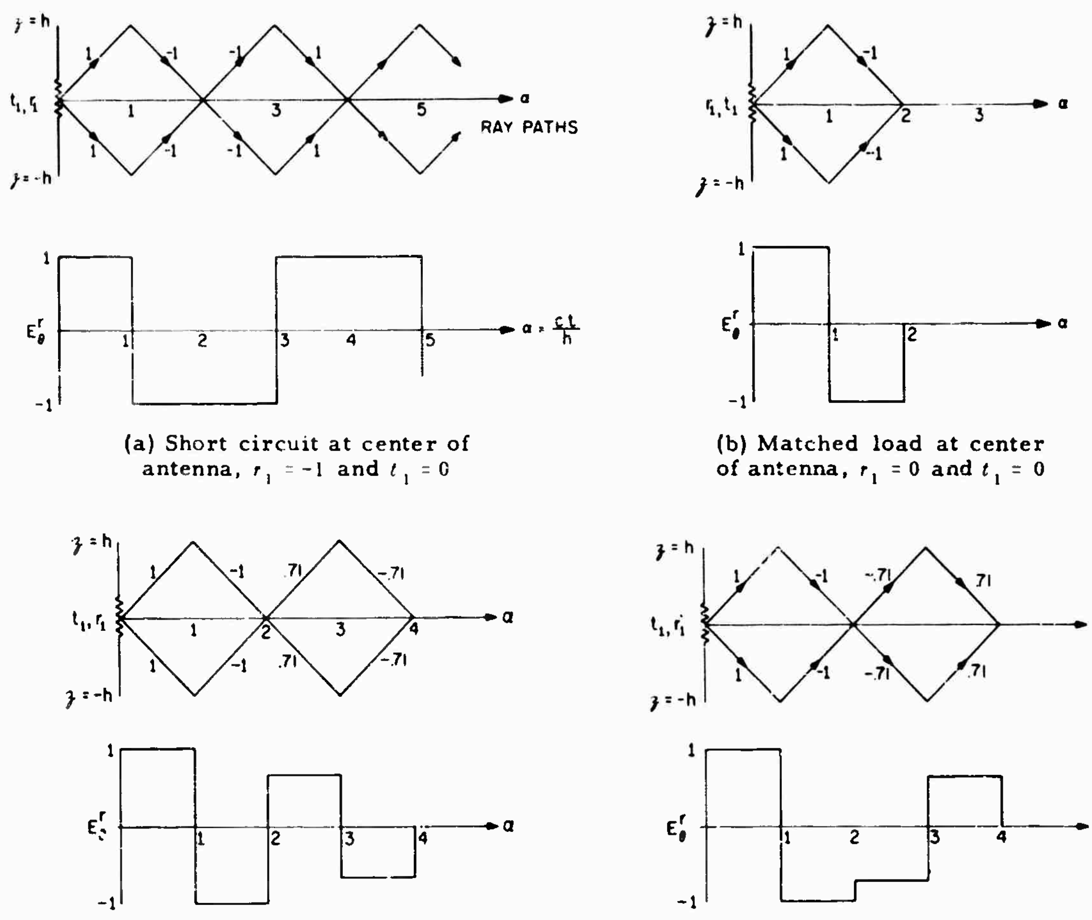


Fig. 5 - Traveling waves and transient electric fields

Figure 5a shows the ray paths and transient electric field for a short-circuited antenna. Since no account is taken here of the antenna impedance, there is no damping of the far field with time. Other transient fields for intermediate cases are shown in Figs. 5b, 5c, and 5d. Note that in the critically damped case the wave only travels to the end and returns to the driving point.

In the preceding analysis, it was assumed that the base current was in the form of a step function. The more common case is when the base voltage is in the form of a step function. The base current in the steady and transient states is then a function of the impedance properties of the antenna. Once the actual base current has been computed, superposition can be used to find the far field. Thus, if $i(u)$ is the current in the frequency domain, then

$$i(u) = V(u)/Z(u) . \tag{18}$$

If $V(t)$ is a step function, then

$$i(t) = \frac{V_0}{2} + \frac{V_0}{\pi}\, Im \int_0^\infty \frac{e^{j\omega t}}{z(\omega)} \frac{d\omega}{\omega} . \tag{19}$$

The superposition integral may then be applied to the $i(t)$ of Eq. (19) using the step function response of Eq. (17):

$$E_\theta^r(t) = i(0)\, E_{\theta s}^r(t) + \int_0^t i'(\tau)\, E_{\theta s}^r(t-\tau)\, d\tau \tag{20}$$

or, using integration by parts,

$$E_\theta^r(t) = i(t)\, E_{\theta s}^r(0) - \int_0^t i(\tau)\, E_{\theta s}^{r\,\prime}(t-\tau)\, d\tau , \tag{21}$$

where $E_{\theta s}^r(t)$ is the step response and the prime denotes differentiation. The superposition given by Eq. (21) is particularly useful for this analysis.

A simple, although crude, approximation for the driving point impedance of a linear antenna is given in Chapter 1, Eqs. (16), as

$$Z_0 \approx \bar{R}_0 + j \sum_{\nu=1}^{\infty} \bar{X}_\nu \cos \nu\beta_0 h \tag{22a}$$

$$\approx \bar{R}_0 + j\bar{X}_1 \cos \beta_0 a + \ldots , \tag{22b}$$

where

$$\bar{R}_0 \approx 150 \text{ ohms} ,$$

$$\bar{X}_\nu = \frac{m}{\pi\nu^2} (\cos \nu\pi - 1) , \qquad m = 500 ,$$

and

$$\Omega = 2 \ln (2h/a) = 15 .$$

From Eqs. (19) and (22b) the current $i(t)$ is

$$i(t) = \frac{V_0}{2} + \frac{V_0}{\pi R}\, Im \int_0^\infty \frac{e^{j\omega t}}{1 + j\xi \cos \frac{\omega h}{c}} \frac{d\omega}{\omega} , \tag{23}$$

where

$$\xi = \frac{X}{R} \begin{cases} R = \bar{R}_0 + R_L , & R_L = \text{load resistance} , \\ X = \bar{X}_1 + X_L , & X_L = \text{load reactance} . \end{cases}$$

The integrand of Eq. (21) has a simple pole at $\omega = 0$ on the real axis and an infinite number of complex poles located at

$$\frac{h}{c} x = \frac{n\pi}{2} , \quad n \text{ odd} , \tag{24}$$

and

$$\sinh \frac{h}{c} y = (-1)^{n+1} \xi^{-1}, \tag{25}$$

where $\omega = x + jy$. The poles of Eqs. (24) and (25) in the upper half plane are at

$$\frac{h}{c} x = (4n+1) \frac{\pi}{2}, \quad n = 0, 1, 2, \ldots; \quad y = \frac{h}{c} \sinh^{-1} \xi^{-1}. \tag{26}$$

With Eq. (26), the transient part of $i(t)$ in Eq. (23) is given by

$$i(t) = \frac{V_0}{\pi R} e^{-(h/c)t \sinh^{-1} \xi^{-1}} \; Im \sum_{n=0}^{\infty} \frac{\left(e^{j(\pi/2)\alpha_1}\right)^{4n+1}}{(4n+1)\frac{\pi}{2} + j\xi^{-1}}, \tag{27}$$

where $\alpha_1 = (h/c)t$. The first and dominant term of Eq. (27) is

$$i(t) \approx \frac{V_0}{2R} e^{-(h/c)t \sinh^{-1} \xi^{-1}} \frac{\sin \frac{\pi}{2} \frac{c}{h} t}{\left(\frac{\pi}{2}\right)^2 + \xi^2}. \tag{28}$$

The sum of the remaining terms is slowly varying except at $\alpha_1 = 1, 5, 9, \ldots$, where it has an infinite peak.* This peak is due to the order of the approximation used for the impedance. Equation (28) is a surprisingly good approximation to the current.

Since $i(0) = 0$, the far field is found by substituting Eq. (28) in Eq. (21), or

$$E_\theta^r(t) = i(t) E_{\theta s}^r(0) - i_0 \int_0^t e^{-(c/h)t \sinh^{-1} \xi^{-1}} \left[-\frac{c}{h} \sin^{-1} \xi^{-1} \sin\left(\frac{\pi}{2} \frac{c}{h} t\right) + \frac{c}{h} \frac{\pi}{2} \cos\left(\frac{\pi}{2} \frac{c}{h} t\right) \right] E_{\theta s}^r(t - \tau)\, d\tau. \tag{29}$$

The derivative of the step voltage response of the electric field $E_{\theta s}^{r\,\prime}$ can be found directly from Eq. (17):

$$E_{\theta s}^{r\,\prime}(t) = \frac{120 I_s(0)}{r} \{[\delta(t) - \delta(t-a)] - [\delta(t-a) - \delta(t-2a)] + (r_1 - t_1)[\delta(t-2a) - \delta(t-3a)] - (r_1 - t_1)[\delta(t-3a) - \delta(t-4a)] + \ldots\}. \tag{30}$$

---

*Since $(4n+1) >> \xi^{-1}$ for $n > 0$, the sum in Eq. (27) is given by the following formula for $n > 0$ (note that the $n = 0$ term is given by Eq. (28)):

$$\overline{i(t)} \approx i_0 e^{-\alpha_1 \sinh^{-1} \xi^{-1}} \; Im \sum_{n=1}^{\infty} \frac{\left(e^{j(\pi/2)\alpha_1}\right)^{4n+1}}{(4n+1)\frac{\pi}{2}},$$

where the bar over $i(t)$ denotes that the $n = 0$ term is missing. It can be shown that $i(t)$ can be given as the addition of two related sums:

$$\overline{i(t)} \approx i_0 e^{-\alpha_1 \sinh^{-1} \xi^{-1}} \left[\sum_{k=1}^{\infty} \frac{\sin (2k-1)\alpha_1}{2k-1} + \sum_{k=1}^{\infty} (-1)^{k-1} \frac{\sin (2k-1)\alpha_1}{2k-1} \right]$$

$$\approx i_0 e^{-a \sinh^{-1} \xi^{-1}} \frac{2}{\pi} \left[\frac{\pi}{4} + \frac{1}{2} \ln \tan \left(\frac{\pi}{4} + \frac{\pi \alpha_1}{4} \right) \right].$$

Substitution of Eq. (30) into Eq. (29) yields

$$E_\theta^r(t) = i(t)\,E_{\theta s}^r(0) - E_{\theta s}^r(0)\int_0^t i(\tau)\,\Big\{\big[\delta(t-\tau) - \delta(t-\tau-a)\big]$$
$$- \big[\delta(t-\tau-a) - \delta(t-\tau-2a)\big] + (r_1 - t_1)\big[\delta(t-\tau-2a) - \delta(t-\tau-3a)\big]$$
$$- (r_1 - t_1)\big[\delta(t-\tau-3a) - \delta(t-\tau-4a)\big] + \ldots\Big\}\,d\tau\,, \tag{31}$$

where

$$i(\tau) = Ae^{-(c/h)t\,\sinh^{-1}\xi^{-1}}\cos\left(\frac{\pi}{2}\,\frac{c}{h}\,\tau - \varphi\right),$$

$$A = \frac{120 I_z(0)}{r}\,\frac{c}{h}\sqrt{\frac{\pi^2}{4} + (\sinh^{-1}\xi^{-1})^2}\,,$$

and

$$\varphi = \tan^{-1}\frac{4\sinh^{-1}\xi^{-1}}{\pi}\,.$$

By direct integration of Eq. (31) $E_\theta^r(t)$ is given entirely in terms of $i(t)$:

$$E_\theta^r(t) = i(t)\,E_\theta^r(0) - E_\theta^r(0)\,\Big\{\big[i(t) - i(t-a)\big] - \big[i(t-a) - i(t-2a)\big]$$
$$+ (r_1 - t_1)\big[i(t-2a) - i(t-3a)\big] - (r_1 - t_1)\big[i(t-3a) - i(t-4a)\big] + \ldots\Big\}\,. \tag{32}$$

A plot of Eq. (32) for various values of r_1 and t_1 is shown in Fig. 6. Note that the initial response ($t < 2a$) is independent of the load impedance.

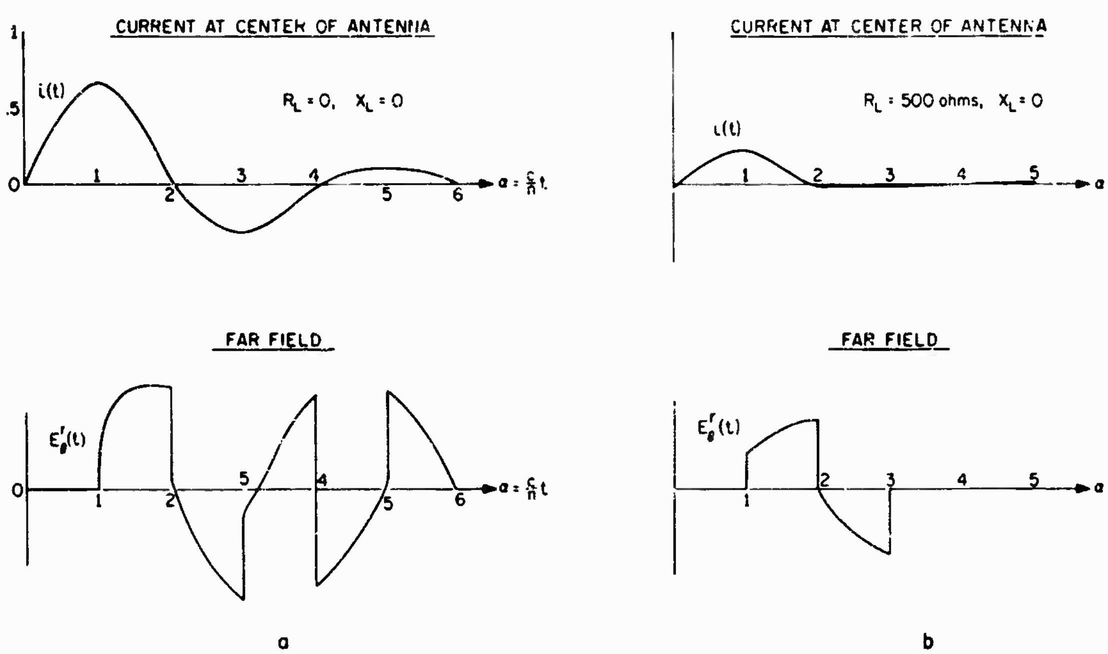


Fig. 6 - Antenna current and far field; (a) $R_L = 0$ and $X_L = 0$, (b) R_L = 500 ohms and $X_L = 0$

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CHAPTER 3

BACKSCATTERED FIELD OF A LINEAR ANTENNA AND A TWO-ELEMENT ARRAY

INTRODUCTION

The two previous chapters have investigated the transient far field and terminal properties of a single linear antenna. The antennas were driven by a generator which produced a step function or a rectangular pulse of voltage or current. In the present chapter, a limited range of impedance data is used to give the transient backscattered field of a linear antenna and a two-element array. The plane wave was considered to have a step function or rectangular pulse behavior as a function of time. In all cases the electric field vector of the incident wave was parallel to the z axis of the linear antennas. The backscattered field was computed only in the direction of the original excitation.

THEORETICAL FORMULATION

The reciprocity relation can be used to find the current I_L through the load resistor of the antenna shown in Figs. 1 and 2. If the magnitude of the incident electric field is E_0^i, then Thevinin's theorem (1) yields

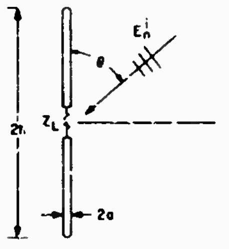
$$I_L = \frac{V_{oc}}{Z_0 + Z_L} = \frac{-2h_e(\theta, \beta_0 h)}{Z_0 + Z_L} E_0^i e^{j\omega\tau_1} , \tag{1}$$

where $h_e(\theta, \beta_0 h)$ = effective half-length of the antenna and $\tau_1 = t - (r/c)$. If $F(\theta, \beta_0 h)$ is the field characteristic on transmission, then the reradiated electric field E^{rr} can be found from a knowledge of the current I_L induced by the original field:

$$E_\theta^{rr} = \frac{j\zeta_0}{2\pi} I_L F(\theta, \beta_0 h) e^{-j\omega\tau_2} . \tag{2}$$

where $\tau_2 = t - (r/c)$. Since only the current at the center of the antenna is involved in the determination of the backscattered field, certain approximations are inherent in the method. The approximation is that the complete backscattered field is not determined; only the symmetrical part of the field is involved in finding currents or voltages across

Fig. 1 - Linear antenna



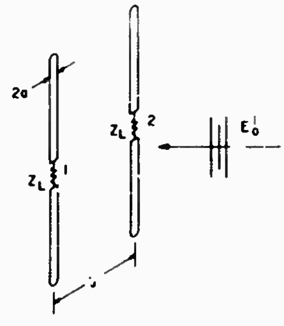


Fig. 2 - Array of two linear antennas

loads connected to the antenna terminals. Thus the backscattered field E_θ^{rr} is more correctly the load current or voltage at the terminals of a perfect dipole placed in the far field of the reradiating antenna. The steady-state effective lengths and impedances, however, are based on the actual symmetrical currents that exist on the antennas (2,3).

Since the field factor $F(\theta, \beta_0 h)$ of the transmitting antenna is related to the effective length, Eq. (2) becomes

$$E_\theta^{rr} = \frac{j\zeta_0}{2\pi} \frac{(-2h_e)}{Z_0 + Z_L} E_\theta^i e^{j\omega\tau_1} \beta_0 h_e e^{-j\omega\tau_2}, \tag{3}$$

where

$$\beta_0 h_e = F(\theta, \beta_0 h). \tag{4}$$

With $a = \tau_1 - \tau_2$, Eq. (3) is reduced to

$$E_\theta^{rr} = \frac{-j\zeta_0}{\pi} E_\theta^i \frac{\beta h_e^2(\theta, \beta_0 h)}{Z_0 + Z_L} e^{j\omega a}, \tag{5}$$

where $a = (c/h)t$. Since the Fourier representation of a step function is given by

$$V(t) = \frac{V_0}{2} + \frac{V_0}{\pi} \int_0^\infty \frac{\sin \omega t}{\omega} d\omega, \tag{6}$$

then the scattered field response to a primary field of step function form is

$$E_\theta^{rr}(t) = \frac{\zeta_0}{\pi r} E_0 \, Im \left[\int_0^\infty \frac{\beta^2 h_e^2}{\beta_0 h \, (Z_0 + Z_L)} \frac{e^{j\omega a}}{\omega \frac{h}{c}} d\frac{\omega h}{c} \right] \tag{7}$$

or, since $\beta_0 h = (\omega/c)h$,

$$E_\theta^{rr}(t) = \frac{\zeta_0}{\pi r} (E_0 h) \, Im \left[\int_0^\infty \mathcal{H}(\beta_0 h) e^{j(\beta_0 h)(ca/h)} d(\beta_0 h) \right], \tag{8}$$

where

$$\mathcal{H} = \frac{\beta^2 h_0^2(\theta, \beta_0 h)}{(\beta_0 h)^2 (Z_0 + Z_L)}.$$

With $x = \beta_0 h$ and $t = ca/h$ the reradiated field can be placed in the form of a Fourier transform, or

$$E_0^{rr}(t) = C_0 \, Im \left[\int_0^\infty \mathcal{H}(x) e^{jxt} dx \right]. \tag{9}$$

The function $\mathcal{H}(x) = \mathcal{H}_r + j\mathcal{H}_i$ is shown in Fig. 2 for two different values of load resistance $Z_i = 0$ and $Z_L = Z_0^*$. This function decreases rapidly for $\beta_0 h = \pi$ with the major region of interest in the range $0 \le \beta_0 h \le \pi$. As a first approximation the higher frequency values of $\mathcal{H}(x)$ are neglected beyond the limited range given in Fig. 2. Because of this it is expected that the fine details of the time response will not be reproduced, and the

response for small t will be in error. However, this latter error does not play an important role, since the leading pulse edges are not straight. The time response was found by first approximating the function $H(x)$ by a series of displaced sinusoids $f(x)$, where

$$f(x) = \begin{cases} 0, & x < x_1, \\ b \sin \frac{\pi}{a}(x - x_1), & x_1 \le x \le x_1 + a, \\ 0, & x > x_1 + a. \end{cases} \tag{10}$$

The series of displaced sinusoids is then substituted in the explicit formula for the reradiated field given by

$$E_\theta^{rr}(t) = C\left[\int_0^\infty H_i(x) \cos xt \, dx + \int_0^\infty H_r(x) \sin xt \, dx\right]. \tag{11}$$

The two integrals involved are $C(u;\ x_i,\ a_i)$ and $S(u;\ x_i,\ a_i)$, where

$$C(u;\ x_i,\ a_i) = \int_0^\infty f(x;\ x_i,\ a_i) \cos ux \, dx \tag{12}$$

and

$$S(u;\ x_i,\ a_i) = \int_0^\infty f(x;\ x_i,\ a_i) \sin ux \, dx, \tag{13}$$

where

$$f(x;\ x_i,\ a_i) = \begin{cases} 0, & x < x_1, \\ \sin \frac{\pi}{a_i}(x - x_i), & x_i \le x \le x_1 + a, \\ 0, & x > x_1 + a. \end{cases} \tag{14}$$

By elementary methods the integrals in Eqs. (12) and (13) yield after integration

$$C(u;\ x_i,\ a_i) = \frac{\pi}{a_i}\left\{\frac{\cos\,[(x_i + a_i)u] + \cos ux}{\left(\frac{\pi}{a_i}\right)^2 - u^2}\right\} \tag{15}$$

and

$$S(u;\ x_i,\ a_i) = \frac{\pi}{a_i}\left\{\frac{\sin\,[(x_i + a_i)u] + \sin ux}{\left(\frac{\pi}{a_i}\right)^2 - u^2}\right\}. \tag{16}$$

In the above equation u is the normalized time variable $(c/h)\tau$. For example the reradiated field of a single antenna with $Z_L = 0$ is given approximately by

$$E_\theta^{rr}(t) \approx C_0 \left[0.26\, C\left(u, \frac{7\pi}{20}, \frac{\pi}{10}\right) - 0.26\, C\left(u, \frac{9\pi}{20}, \frac{4\pi}{10}\right)\right.$$

$$\left. + 0.42\, S\left(u, \frac{4\pi}{10}, \frac{2\pi}{10}\right) - 0.17\, S\left(u, \frac{7\pi}{10}, \frac{3\pi}{10}\right)\right]. \tag{17}$$

The approximate step and pulse response for $Z_L = 0$ and $Z_L = Z_0^*$ is given in Fig. 3. It may be concluded from these results that the time behavior is insensitive to the pulse length. The result of changing the load resistance from $Z_L = 0$ to $Z_L = Z_0^*$ is hardly detectable.

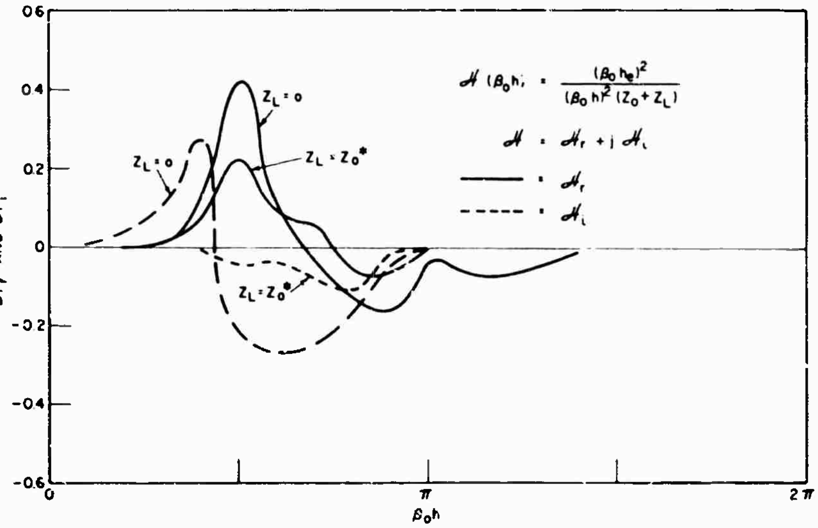


Fig. 3 - Effect of load impedance on $\mathcal{H}(\beta_0 h)$

TRANSIENT BACKSCATTER FROM A TWO-ELEMENT ARRAY

The effective voltage at the receiving terminals of an element in an array concerns not only the effective length of the element but also the array factor. Thus the equivalent voltage at the receiving terminals of an array is

$$V_{oc} \approx -2h\, \frac{(\beta_0 h_e)^2\, A(\theta,\phi)}{Z_{in} + Z_L}, \tag{18}$$

where Z_{in} is the driving point impedance of an element in the array and $A(\theta,\phi)$ is the array factor. The array factor $A(\theta,\phi)$ for an N-element array is

$$A(\theta,\phi) = \frac{\sin Nx}{N \sin x}, \tag{19}$$

where

N = total number of elements

$$x = \pi\left(\frac{b}{\lambda} \cos\phi \sin\theta - \frac{\delta}{\lambda}\right)$$

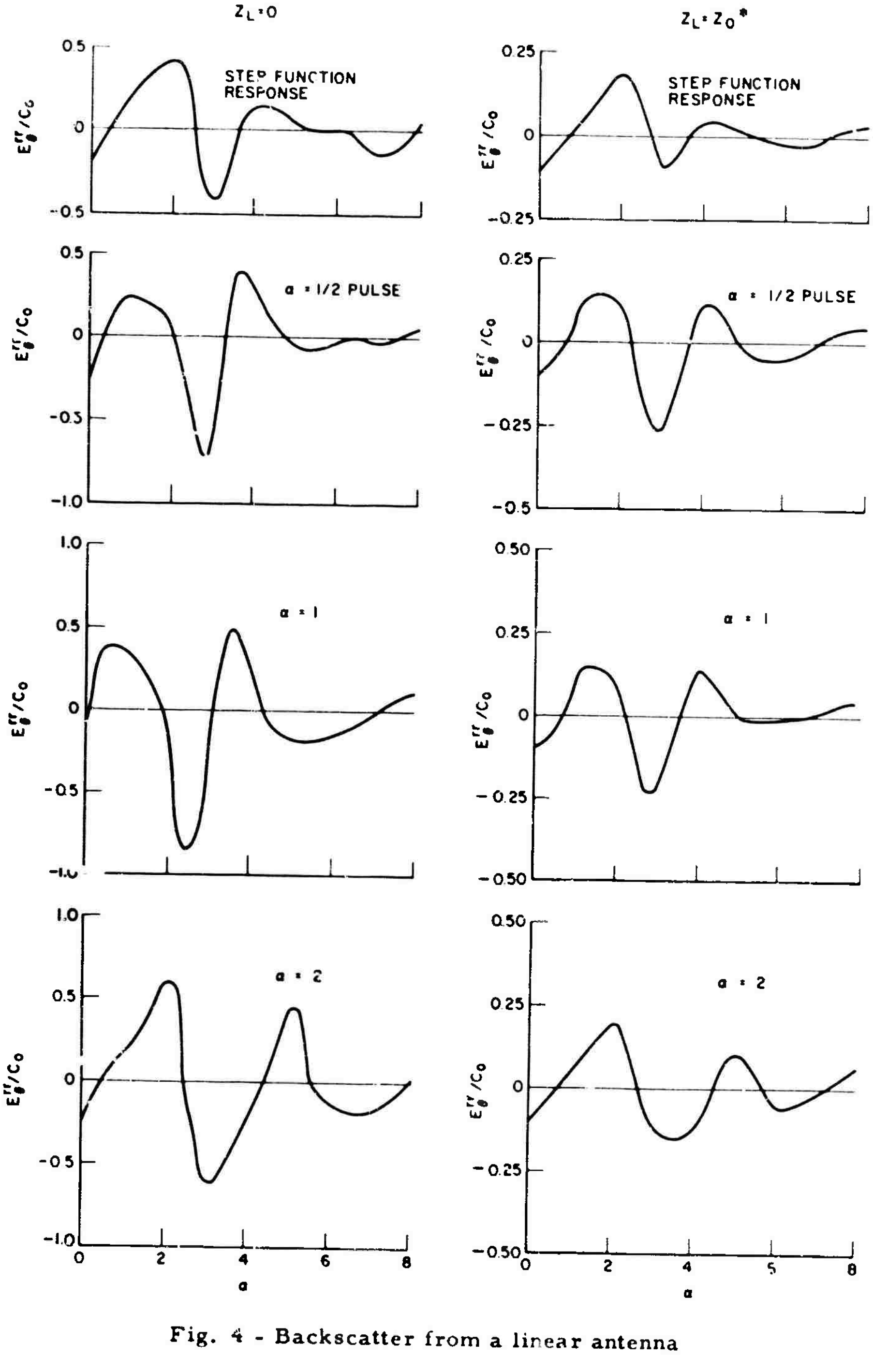


Fig. 4 - Backscatter from a linear antenna

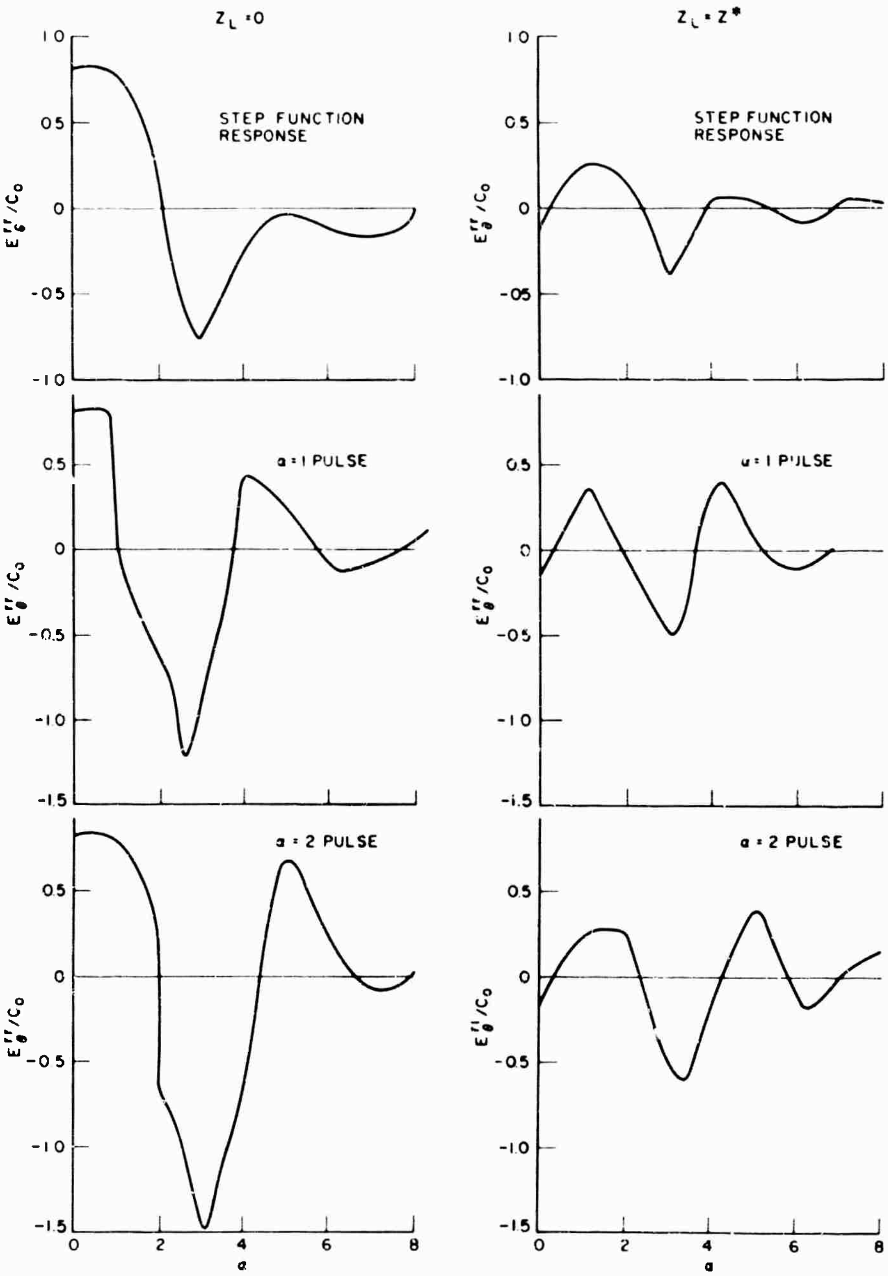


Fig. 5 - Backscatter from a two-element array

b = element spacing

δ = phase delay between elements.

For a two-element array in the H plane ($\theta = \pi/2$),

$$A(\theta,\phi) = \frac{2 \sin(\beta b \cos \phi)}{\sin\left(\frac{\beta b}{2} \cos \phi\right)} - \cos\left(\frac{\beta b}{2} \cos \phi\right) \tag{20}$$

The function $H(x)$ corresponding to Eq. (18) is shown in Fig. 4 calculated from the King-Sandler quasi-zeroth-order impedances. Since driving-point impedances for arrays were calculated only for $\beta_0 b \leq \pi$ and $\beta_0 h \leq \pi$, only the restricted range which included the first antiresonance was used.

Note that as the frequency changes, both the element length and the separation must be increased in the same ratio.

The same method of performing the Fourier integration was used for arrays as for the single antenna. The step and pulse field response is shown in Fig. 5 for the two cases $Z_L = Z_0^*$ and $Z_L = 0$.

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13. ABSTRACT

Three papers entitled "Transient Response of a Linear Antenna to Pulse Voltage Excitation," "Transient Terminal and Field Properties of Linear Antennas," and "Backscattered Field of a Linear Antenna and a Two Element Array" constitute the three chapters of this report. They represent the approach taken by Dr. Sandler to the analysis of scattering from an array of linear antennas. The first paper develops an expression for the far field of a linear antenna excited by a pulse. A simple approximation for the current distribution on the antenna is assumed in this derivation. The second paper deals with essentially the same problem only from a point of view which is analogous to traveling waves on a transmission line. The concepts developed in these two papers have been incorporated into the third to investigate the backscatter properties of both a linear antenna and an array composed of two linear elements.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Linear antenna Antenna array Transient response Linear antenna backscatter Array backscatter Induced current on antenna Antenna impedance						

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